Hybrid Parameters for Common-Emitter Amps

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What I am going to try to do is explain what hybrid parameters are and how to use them.

First of all, breaking it down, you get hybrid, which means a mixture, and parameters, which means a constant with variable values. In the case of hybrid parameters we mean a mixture of constants (some measured in ohms, some in mhos, and some in pure numbers) in which a particular transistor determines the values. Due to the limited size of this paper I am only going to deal with transistors in the common emitter mode of operation.

The Hybrid parameter (h-parameter) approach is one of the most accurate methods which takes all transistor and resistor characteristics into account to calculate small signal current gain (Ai), small signal voltage gain (Av), input impedance (Zin), and output impedance (Zout). Output impedance tells you what load impedance will produce the most power gain (Ap--which is voltage gain times current gain).

To start off with we have h_{ie} which is the base to emitter dynamic resistance and is measured in ohms. It is measured by varying the base to emitter voltage (V_{BE}) and measuring the change in base current (I_B) with the collector-to emitter voltage (V_{CE}) held constant. Then we divide the change (or delta) in base to emitter voltage by delta base current which gives us:

$$hie = \frac{\Delta V_{BE}}{\Delta I_B} \quad \text{with} \quad V_{CE} = C$$

Next we have h_{fe} which is the forward current transfer ratio in a common emitter (CE) stage and is measured as a pure number. Hfe is measured by varying the base current and measuring the change in collector current (I_C) with collector to emitter voltage held constant. Then we divide delta collector current by delta base current which yields:

$$h_{fe} = \frac{\Delta I_c}{\Delta I_B} \quad \text{with} \quad V_{CE} = C$$

Next we have hre which is the reverse voltage transfer ratio of a common emitter stage and is measured as a pure number. We measure hre by changing the collector to emitter voltage and measuring the change in base to emitter voltage (V_{BE}) with base current held constant. We then divide delta base to emitter voltage by delta collector to emitter voltage which gives us:

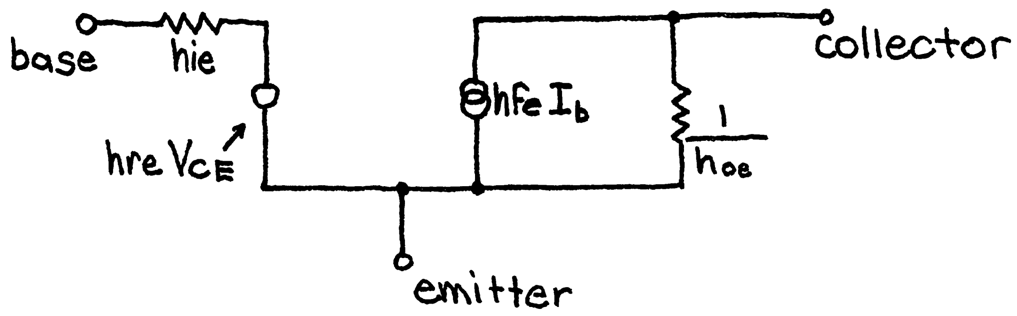
$$h_{re} = \frac{\Delta V_{BE}}{\Delta V_{CE}} \quad \text{with} \quad I_B = C$$

And last of all we have hoe which is the collector to emitter dynamic conductance and is measured in mhos. It is measured by changing the collector to emitter voltage and measuring the change in collector current with the base current held constant. Then divide delta collector current by delta collector to emitter voltage which gives us:

$$h_{oe} = \frac{\Delta I_c}{\Delta V_{CE}} \quad \text{with} \quad I_B = C$$

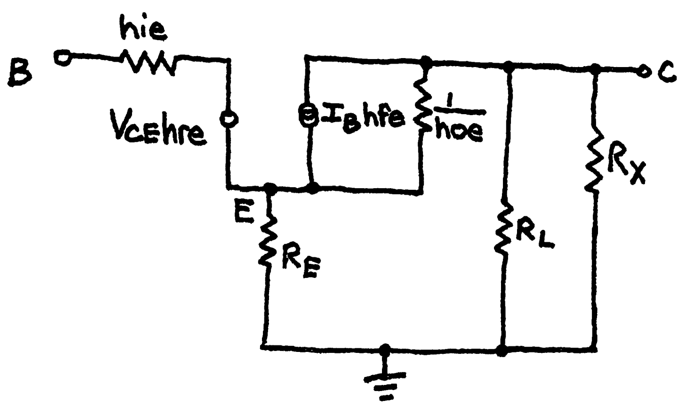
The reason for holding the collector to emitter voltage and the base current constant is so that when you are making a test for a particular parameter, none of the others will interfere.

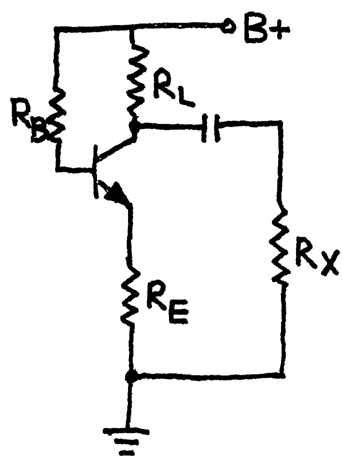
The next thing we need to know is the h-parameter common emitter equivalent circuit which is shown on the next page.



Utilizing the h-parameters, the h-parameter common emitter equivalent circuit, ohms and watts law, and the formula for parallel resistors, we shall solve for Ai, Av, Zin and Zout.

The following diagrams are the common emitter equivalent circuit with associated components as compared with the actual schematic diagram. Note that the load resistor (R_L) and the input impedance of the following stage (R_X) are in parallel and to ground on the h-parameter equivalent circuit. This is because of the filter capacitor in your power supply which puts B-plus and ground at the same level (AC short).





Current gain is equal to the change in output current (I_{out}) divided by the change in input current (I_{IN}) which gives us:

$$A_i = \frac{\Delta I_{OUT}}{\Delta I_{in}}$$

The output current is the current through the input of the next stage. The input current is the current through hie. Let's assume that the change in input current is 1, which means that current gain is equal to the change in output current. To solve for output current we must multiply input current (which is one) by negative hfe which will give us total current through hoe, R_L, R_X and R_E. We multiply it by negative hfe because a common emitter amplifier inverts. By using ohms law and what has been explained one can find current through R_X as follows:

$$\Delta I_X = \Delta I_{OUT} = A_i =$$

$$\frac{-h_{fe}\left[\dfrac{\dfrac{1}{h_{oe}}\left(\dfrac{R_L R_X}{R_L + R_X} + R_E\right)}{\dfrac{1}{h_{oe}} + \dfrac{R_L R_X}{R_L + R_X} + R_E}\right]\left[\dfrac{R_L R_X}{R_L + R_X}\right]}{\left[\dfrac{R_L R_X}{R_L + R_X} + R_E\right] R_X}$$

which simplifies into:

$$\underset{(\text{into } R_X)}{A_i} = \frac{-h_{fe} R_L}{R_L(1 + R_E h_{oe}) + R_X(1 + R_L h_{oe} + R_E h_{oe})}$$

Next we have voltage gain which is equal to the change in output voltage (V_{OUT}) divided by the change in input voltage (V_{IN}) which gives us:

$$A_V = \frac{\Delta V_{OUT}}{\Delta V_{in}}$$

The input voltage is the voltage from the base to ground. The output voltage is the voltage across R_L and R_X. Since R_L and R_X are in parallel, therefore having the same voltage across them, we shall treat them as one resistor called R_{LX}, where:

$$R_{LX} = \frac{R_L R_X}{R_L + R_X}$$

Solving for the change in output voltage and assuming that change in base current equals one, we get:

$$\Delta V_{OUT} = \frac{-hfe\left[\dfrac{\frac{1}{hoe}(R_{LX}+R_E)}{\frac{1}{hoe}+R_{LX}+R_E}\right] R_{LX}}{R_{LX} + R_E}$$

To solve for change in input voltage, assuming change in input current is one, you must add the voltage drop across hie with the voltage drop across hre, which is negative because V_{CE} is negative, with the voltage drop across R_E because they are all in series. Remember, you have collector current and base current flowing through R_E. Solving for change in input voltage yields:

$$\Delta V_{in} =$$

$$hie + \frac{-hfe\frac{1}{hoe}(R_{LX}+R_E)\,hre}{\frac{1}{hoe}+R_{LX}+R_E} + \left[\frac{hfe\frac{1}{hoe}(R_{LX}+R_E)}{(\frac{1}{hoe}+R_{LX}+R_E)(R_{LX}+R_E)} + 1\right] R_E$$

Note that you don't use a negative sign before the hfe when you are referring to R_E because the voltage across R_E is not inverted. Solving for voltage gain and simplifying gives:

$$A_V = \frac{-h_{fe} R_{LX}}{R_{LX}\Delta h + R_E(h_{fe} + \Delta h + R_{LX}h_{oe} + R_E h_{oe} + 1) + h_{ie}}$$

where: $\Delta h = h_{ie}h_{oe} - h_{fe}h_{re}$.

Solving for the emitter resistor in terms of voltage gain is a useful formula to determine what emitter resistor to use for a given voltage gain. Note that if the answer is negative that stage cannot have that high a voltage gain. Also note the term $R_E$hoe on the right side; this is so small a number that it has almost no effect on the answer and can be taken out. Solving for R_E in terms of Av gives:

$$R_E = \frac{h_{fe} + A_V R_{LX}\Delta h + h_{ie}A_V}{A_V(h_{fe} + \Delta h + R_{LX}h_{oe} + R_E h_{oe} + 1)}$$

Remember that Av is a negative number.

The input impedance is the impedance from the base to ground, and is equal to the change in input voltage divided by the change in input current. Assuming the change in input current is one, then the input impedance equals the change in input voltage. Having already solved for the change in input voltage, if we simplify we get:

$$Z_{in} = \frac{R_E\left(h_{ie} + R_E - h_{fe}h_{re}\frac{1}{h_{oe}} + h_{fe}\frac{1}{h_{oe}} + R_{LX} + \frac{1}{h_{oe}}(h_{ie} - h_{fe}R_{LX}h_{re}) + R_{LX}h_{ie}\right.}{R_{LX} + R_E + \frac{1}{h_{oe}}}$$

To get the output impedance, or to figure out what $\underline{R_{LX}}$ will have maximum power gain, multiply the $\underline{A_V}$ formula by the $\underline{A_I}$ formula and take the first derivative; then solve for $\underline{R_{LX}}$ with the top half of the formula equaling zero, which gives:

$$R_{LX} = \sqrt{\frac{R_E X + h_{ie} + R_E^2 h_{oe} X + R_E h_{oe} h_{ie}}{h_{oe} \Delta h}}$$

where: $X = h_{FE} + \Delta h + h_{oe} R_{LX} + R_E h_{oe} + 1$

Note the term $\underline{R_{LX} hoe}$ in $\underline{X}$; again this is so small that it will not affect the answer so it may be taken out.

These formulas are the most accurate way to determine the performance of a given circuit and are very useful in designing electronic equipment using bipolar transistors.